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Stiffness and Angular Deflection analysis of Revolute Manipulator

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Abstract:
This paper proposed to determine the Cartesian stiffness matrix and angular deflection analysis of revolute manipulator. The selected manipulator has rigid fixed link, two movable links and two rotary joints with joint stiffness coefficients are taken into account. The kinematic model of revolute joint manipulator has considered as a planar kinematic chain, which is composed by rigid fixed link and two revolute joints with clearance and deformable elements. The calculation of stiffness matrix depends on Jacobian matrix and change of configuration. The rotational joints are modeled as torsion springs with the same stiffness constant. The relative angular deflections are proportional to the actuated torques taken into account. The subject of this paper has to describe a method for stiffness analysis of serial manipulator. In the present work is to derive the stiffness matrix and angular deflection equations in the Robotic manipulator under the consideration of two-link optimum geometry model for rotary joint manipulator. The stiffness values are measured by displacements of its revolute links loaded by force.
Keywords: Angular deflection, Jacobian matrix, Robotic manipulator, Rotary joint, Stiffness matrix.

I. Introduction:

The design procedure of revolute manipulator becomes highly iterative. Present days, computer based industries are designed to manufacture identical products and variety of goods in a limited number of batches as per the required design, material components, tooling and depends on fluctuations in market demand. Therefore the design processing of new roles are developing for Robotics. Li, Wang, Lung-wen Tsai and Merlet [1,2,3] are analyzed the stiffness approach, is used for the calculation of stiffness analysis of a Stewart platform. Huang, Zhao, White house, Khasawnesh and Ferreira [4, 5] are the stiffness technique analysis for comparative studies. Griffis and Duffy [6] discussed the asymmetric nature of Cartesian stiffness matrix. Tsai, Joshi, Carricato and Parenti Castelli [7, 8] dealt with serial manipulators and considered only the actuated joints. Fresonke, Hernandez, Tesar and Paul [9, 10] have set analytical criteria for the deflection prediction of serial manipulators. Mark, Craig, Deb and Duffy [11, 12, 13, 14] are discussed the Jacobian matrix. Pathak, Niku, Schilling, Fu and Saha[15,16,17,18,19] are discussed the velocity analysis. The kinematic analysis of a robot has composed by rigid link which is connected together with rotary elastic joints. The joints are considered as very simple, such as revolute joints and assumed all joints have only single degree of freedom. This revolutionary change in the Robotic technology has taken this field to different positions. In the present work is to derive the stiffness matrix and angular deflection equations in the robotic manipulator under the consideration of two-link optimum geometry model for rotary joint manipulator. Consider the optimum geometry of a regional two link structural model for a manipulator with only revolute joints, has the first limb is vertical, the second intersecting it orthogonally and the third parallel to the second limb as shown in Fig (1). In Fig (1) the joint J_{12} considered at the shoulder rotation and joint J_{23} considered at the elbow rotation. The three wrist joints are considered at the end of the link 3. The out board end of the link 3 contributes very little static deflection because the bending moments are low and because the cantilever length outboard of a given cross section is small. The wrist joints are modeled as being rigid, when studying static deflection. The error will result from ignoring the wrist and changes in cross section outboard of the wrist, therefore considered outboard as uniform cross section. Any weight due to the wrist actuators, over and above that which would be present if the link 3 of uniform cross section similar to that at its inboard end is lumped with the load and considered to be applied at the global reference point. Similarly the wrist joints contribute little to the lower frequency vibration modes of the system, since the inertia to which they are subjected is relatively low. Thus it will be assumed that the model of the Fig (1) can be applied as the preliminary design model for static cases.

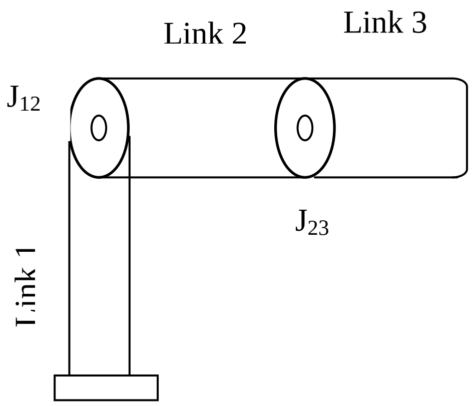

Fig (1): Two link structural model rotary joint manipulator

II. Kinematic analysis of Angular Deflection:

Link 2 and link 3 are assumed to have uniform cross sections. The loading pattern of structural model is as shown in Fig (2). The bending deflections in all members are much larger than the deflections in the longitudinal direction. Thus, the deflection of the pylon due to the bending moment at the inboard end of link 2 produce a significant horizontal displacement of the global reference point, but negligible vertical displacement. However, the angulations of the pylon at joint J_{23} is significant, since the angulations of the pylon are multiplied by the length of the arm and it produces a significant vertical displacement with respect to the reference point.

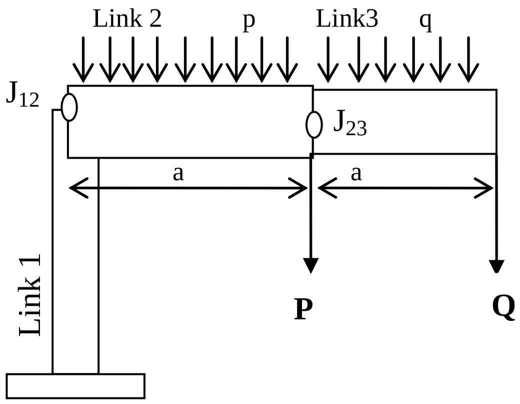

Fig (2) Loading pattern of structural model

When the loading as shown in Fig (2), with 'p' representing the self weight per unit length of link 2, 'q' represents self weight per unit length of link 3, '**P**' represents the weight of the actuators at the elbow, and '**Q**' represents the weight of the actuators and load lumped at the hand.

The bending moment at the joint J_{23} is $M_{23} = aQ + \frac{a^2q}{2}$
Where 'a' is the length of link 3
The Shear force (S) at the same section is $S_{23} = P + Q + aq$
At the shoulder, the bending moment is
$M_{12} = 2aQ + aP + \frac{3a^2q}{2} + \frac{a^2p}{2}$
Hence using beam deflection formulae, the angular deflection of the pylon at the shoulder is
$\phi_{12} = M_{12}.h/EI_p$
Where I_p is the cross sectional second moment of area of the pylon. Assume that the link to be uniform cantilever beam, and 'h' is its height. The additional significant is that angular deflection at this location is caused by deflection of the bearings of joints J_{12}. This is usually the only location in this geometry at which the bearing compliance must be considered. The geometry necessary for computation of bearing deflection is shown in Fig (3).

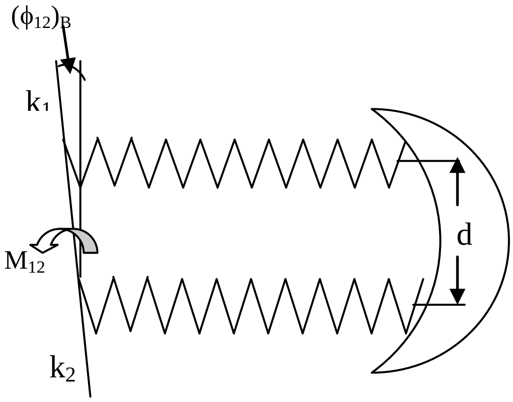

Fig (3): Bearing deflection model.

The springs represents the radial compliance of two rolling element bearings separated by distance 'd', M_{12} is the bending moment at the 'shoulder' and $(\phi_{12})_B$ is the resulting angular deflection due to bearing compliance. k_1 *and* k_2 are the radial stiffness of the upper and lower bearings respectively and'd' is the separation along the joint axis of the bearing planes.

The resulting angular deflection of bearing is
$(\phi_{12})_B = 16M_{12}/d^2 * (1/k_1 + 1/k_2)$
The total angular deflection at the "shoulder" is then

$$\phi_{12} = \frac{1}{EI}\left(2aQ + aP + \frac{3a^2q}{2} + \frac{a^2p}{2}\right)a + \frac{1}{EI}\left(aQ + \frac{a^2q}{2}\right)a$$

$$\Rightarrow \phi_{12} = \frac{a^2}{2EI}(2P + 6Q + ap + 4aq)$$

Vertical deflection of link 2 at joint J_{12} is

$$\delta_{12} = \frac{1}{EI}\left(\frac{a^3P}{3} + a^3Q + \frac{a^4p}{8} + \frac{a^4q}{2}\right)$$

$$\Rightarrow \delta_{12} = \frac{a^3}{24EI}(8P + 24Q + 3ap + 12aq)$$

The total angular deflection at the "elbow" is then

$$\phi_{23} = \frac{M_{23}}{EI}.a = \frac{a^2}{EI}(Q + aq/2)$$

Vertical deflection of the link 2 at joint J_{23} is

$$\delta_{23} = \frac{1}{EI}\left(\frac{a^3 Q}{3} + \frac{a^4 q}{8}\right)$$

$$\Rightarrow \delta_{23} = \frac{a^3}{24EI}(8Q + 3aq)$$

Where 'I', is the second moment of area of the cross section at the elbow

The total deflection at the outboard end is
$Y = \delta_{12} + \delta_{23} + 2a\phi_{12} + a\phi_{23}$

This equation can be used to compute the deflection of the given structural cross section and loads.

III. Stiffness matrix formulation:

Kinematic model of revolute manipulator is considered as serial kinematic chain with two revolute joints. The Jacobian matrix of manipulator is

$$J = \left[\frac{\partial P_x}{\partial \phi_i} \quad \frac{\partial P_y}{\partial \phi_i} \quad \frac{\partial P_{xy}}{\partial \phi_i} \quad \frac{\partial Q_x}{\partial \phi_i} \quad \frac{\partial Q_y}{\partial \phi_i} \quad \frac{\partial Q_{xy}}{\partial \phi_i}\right]^T$$

Where P_x, P_y, Q_x, Q_y we're end coordinates of the rotating limbs and P_{xy}, Q_{xy} are the angular coordinates of the end effectors of the rotating limbs of the revolute manipulator.
ϕ_i is angular coordinate of the revolute joint i
The limbs of the revolute manipulator are assumed as elastic bodies. The joint stiffness is represented with linear torsion spring, along with control loop stiffness of rotary variable differential transducer and actuations are taken into account.
In case of small elastic deformation, the relation between torque and stiffness of joints are $T_i = k_i \delta\phi_i$
Where T_i is the torque applied to the joint
$\delta\phi_i$ is the torsion deformation and
k_i is the joint stiffness value.
Assuming that frictional forces at the joints are neglected and also neglect the gravitational effect, then the relationship between end effectors forces and joints torques are with respect to the principle of virtual work is $T_i = J^T F$ and $k_i = {T_i}^{-1}\delta\phi_i$
Where F= end effectors output forces and moment vector components are described with respect to the base fixed coordinate system. Then the stiffness values are measured by the displacements of its end effectors are loaded by forces.

IV. Conclusion:

This paper proposed to calculate the relative angular deflections of simple revolute manipulator with only revolute joints and proportional to the actuated torques which are measured by using rotary variable differential transducer. And also to determine the Cartesian stiffness matrices which are measured by the displacements of its revolute limbs loaded by forces. The moving limbs are modified as linear springs with the same stiffness constant. Similarly all the rotational joints are modeled as torsion springs with the same stiffness constant. In the present work is to compute the stiffness and angular deflection equations in the revolute manipulator and studied the dependence of these quantities on other manipulator parameters. Since the revolute manipulator contains serial chain and has to describe the stiffness matrix which is depends on Jacobian matrix and change of configuration. It is a hope that designers will find these results useful when they are faced with problems of design aspects in manipulators. This present work can be extended by formulating these problems using computer aided engineering tools and analyze the results accordingly.

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